Core Hardenability Calculations for Carburizing Steels

J. M. TARTAGLIA and G. T. ELDIS

Analytical expressions are presented which allow the calculation of an ideal critical diameter (D_I) and a Jominy end-quench hardenability curve for a steel from its chemical composition and prior austenite grain size. The expressions are based on alloy hardenability factors in the literature and on the previously unpublished "hardness drop" method of determining D_I from end-quench hardenability curves. Relationships defining Jominy curve shape as a function of D_I are developed. These differ from similar relationships previously published by recognizing that, for steels of low to medium hardenability, the microstructure contains significant amounts of non-martensitic transformation products even at the prescribed first position of hardness measurement on the end-quench hardenability bar, 1.59 mm (1/16 inch) from the quenched end. The analytical expressions presented are particularly well suited for the calculation of D_I and end-quench hardenability curves for boron-free carburizing steels containing 0.15 to 0.25 pct carbon.

I. INTRODUCTION

FOR any steel used in the quenched or quenched and tempered condition, hardenability is one of its most important properties. It is this factor which determines which microconstituents will form on quenching a piece of given size in a given quenchant, and hence determines the final mechanical properties of the heat treated material. The ability to predict hardenability from chemical composition and to calculate Jominy end-quench curves is of great benefit to the materials engineer who wishes to know if a proposed new steel or a given heat of steel can meet the hardenability requirements of a given application. The engineer is often interested in the level of hardness he can obtain in a specific location on a certain part subjected to a given quench, and he often knows the "Jominy Equivalent Cooling Rate" of the location in question, *i.e.*, the position on the standard Jominy end-quench bar which has the same cooling rate as the part location of interest.[1]

A commonly used index of hardenability is the ideal critical diameter (D_I). This is the diameter of a cylindrical steel bar which will form 50 pct martensite at its center when subjected to an ideal quench. An ideal quench is one in which the temperature of the surface of the piece is instantaneously lowered to the temperature of the quenchant, on immersion therein, so that the cooling rate is controlled solely by the thermal diffusivity of the material.

The purpose of this paper is to describe a new method of determining D_I values experimentally using a standard Jominy test and to present regression equations for calculating both D_I and Jominy curves from composition. It should be mentioned at the outset that these regression equations are valid only for calculating the core hardenability of boron-free carburizing steels with 0.15 to 0.25 pct* carbon.

*All alloy contents in this paper are in weight percent.

II. BACKGROUND

The early experiments on quenching that led to the development of the ideal critical diameter concept have been presented by Grossmann.[2] D_I is dependent on the composition and grain size of the steel, and various empirical formulae have been proposed to express this interrelationship. Grossmann[3] was among the first to publish such empirical formulae; he used the model that D_I is equal to some base value (dependent only on carbon content and grain size) times various multiplying factors for the different alloying elements. Grossmann's work was based on data for steels with 0.6 pct carbon.[3] Using this same model, DeRetana and Doane[4] have determined the multiplying factors for lower carbon steels (0.15 to 0.25 pct C). Their revised alloy multiplying factors[5] and their extension[4,5] of the original base D_I values first published by Kramer *et al.*[6] are shown in Figures 1 and 2, respectively. A comparison of the various methods for predicting D_I values was published

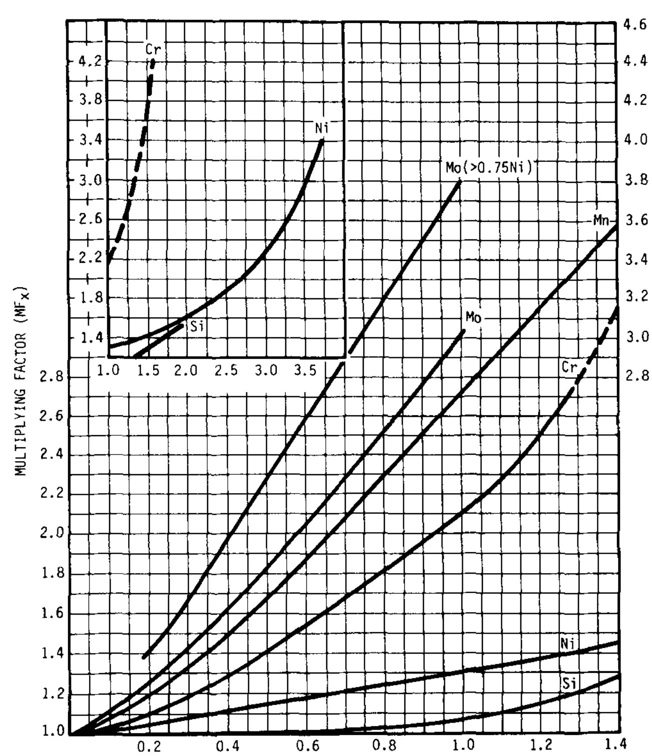

Fig. 1 — Average multiplying factors for several elements in alloy steels containing 0.15 to 0.25 pct carbon (Ref. 5).

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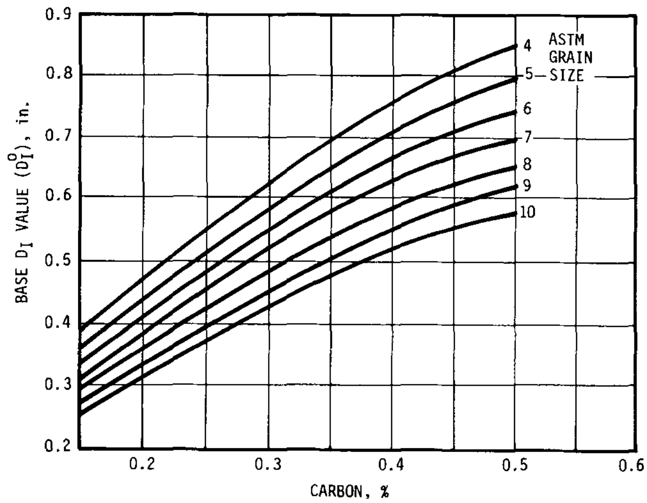


Fig. 2—Multiplying factor for carbon plus grain size (Refs. 4 and 5).

in 1978 by Doane[7] who demonstrated that the DeRetana and Doane factors[4,5] were the most effective available for calculating the ideal critical diameters (D_I) of low carbon steels (0.15 to 0.25 pct C) with low, medium, and high hardenability values.

Actual calculation of the Jominy curve from the D_I value was made possible by the work of Field[8] and Boyd and Field.[9] For 14 different steels, these authors determined the ratio of the initial or quenched-end hardness of the Jominy bar, *IH*, to the hardness at other positions along the bar, *DH*, and tabulated these *IH*/*DH* ratios as a function of the calculated D_I values for the various steels. With these tabulated values, and knowing both the D_I of the steel and the fully martensitic or quenched-end hardness of the material, it is possible to generate the Jominy curve for a steel of a given D_I by dividing the quenched-end hardness by the *IH*/*DH* ratio for each location along the Jominy bar.

The data of Boyd and Field[9] have been incorporated with alloy multiplying factors into several different hardenability calculators. These are essentially a form of circular slide rule which allow the calculation of D_I and the Jominy curve from steel composition and grain size. Such calculators have been issued by the Bethlehem Steel Company, United States Steel Company, and the Climax Molybdenum Company, a subsidiary of AMAX Inc. The Climax device is aimed specifically at the core and case hardenability of carburizing steels. The original Boyd and Field data only covered D_I values ranging from 38 to 185 mm (1.5 to 7.3 inches), and Jominy positions from 6.4 to 50.8 mm (0.25 to 2 inches) at intervals of 6.4 mm (0.25 inch). Since, however, lower D_I values and positions closer to the quenched end of the Jominy bar are often of interest in carburizing steels, the Climax hardenability calculator includes an extrapolation of the Boyd and Field data to lower D_I and smaller J_D* values.

*J_D = Jominy distance. When written with a numerical subscript in place of the subscript *D* (Jominy position), it refers to that number of sixteenths of an inch from the quenched end of the bar. For example, J_1 = 1/16 inch (1.59 mm).

There is now some question regarding the applicability of the Boyd and Field data to carburizing steels (steels with about 0.2 pct C). In working with medium to high hardenability carburizing grades, the present authors have noted that the Boyd and Field data predict a Jominy curve which is consistently lower than the experimental curve at large J_D values. That is, at these larger values of J_D, the *IH*/*DH* ratios of Boyd and Field appear to be too large. Also, the authors have found that the extrapolation of the Boyd and Field data as used on the Climax hardenability calculator may be in substantial error for $J_D < J_4$ when $D_I < 64$ mm (2.5 inches). Sponzilli, *et al.*[10] published a new set of *IH*/*DH* *vs* D_I values in 1975. These are essentially the Boyd and Field data with an improvement in the region of low D_I values and small J_D values based on some additional measurements. The present authors believe these newer *IH*/*DH* ratios are still in substantial error where carburizing steels are concerned. Therefore, the purpose of this paper is to present a body of equations which will calculate the Jominy curves of carburizing steels with greater accuracy.

III. EXPERIMENTAL PROCEDURES

Thirty-two steels covering a calculated D_I range of 21 to 85 mm (0.85 to 3.33 inches) were used for this investigation. These had been prepared as air-induction melted and aluminum-deoxidized heats for another study. Details of preparation and processing to 31.4 mm (1.25 inches) diameter bar stock are given elsewhere.[11] Table I gives the chemical analyses. The prior austenite grain size range of the steels was ASTM No. 8-9 as revealed by the McQuaid-Ehn test.[11] This test of prior austenite grain size was deemed most appropriate for this study of carburizing steels since it involved actual carburization of the test specimens at a typical commercial carburizing temperature (see ASTM Standard E112).

Standard Jominy end-quench specimens were machined from normalized bar stock. These were austenitized at 927 °C (1700 °F) and end-quenched in accordance with SAE Standard J406. After quenching, parallel flats were surface ground to a depth of 0.76 mm (0.03 inch) along the sides of the bar. This depth, in excess of the specified minimum, was necessary to produce a flat of sufficient width for the hardness tests performed in this study.

Both Vickers (5 kg load) and Rockwell C hardness tests were performed on each flat of each bar. Figure 3 shows schematically the arrangement of hardness impressions on the flat. In making the hardness measurements, each Jominy bar was held in a special fixture equipped with a screw drive for moving the bar by measured amounts in a direction parallel to its axis. The Vickers impressions were made first, two sets per flat, along lines located midway between the center of the flat and the edge, and to a minimum distance of $J_D = J_2$. The inter-impression spacing was kept at 0.53 mm (0.02 inch), or 1/3 of the normal Rockwell impression spacing. Then the Rockwell hardness impressions were made in the usual manner along the center line of the flat. The HV5 readings were converted to HRC values by means of a calibration curve, which was determined by measuring the 5 kg Vickers hardness of a series of standard Rockwell test blocks. All hardness data were then plotted in standard Jominy curve form, as shown in Figures 4 and 5 for two steels from the data set.

The D_I values for the various steels were determined from the Jominy curves using Carney's data,[12] reproduced here in Figure 6. This relates the cooling rates at the centers of cylinders of various diameters, subjected to an ideal quench,

Table I. Compositions of the Alloys Studied[a]

Heat Number	Element, Wt Pct					
	C	Mn	Si	Ni	Cr	Mo
5164A	0.22	0.56	0.36	1.45	—[b]	—
5170B	0.21	1.11	0.41	0.74	0.51	0.26
5173C	0.22	0.84	0.38	0.73	1.01	0.25
5175B	0.21	1.08	0.37	1.39	0.51	—
5175C	0.21	1.06	0.37	1.39	0.96	—
5177A	0.21	0.56	0.37	—	—	0.25
5177B	0.21	0.56	0.37	—	0.52	0.25
5177C	0.21	0.56	0.37	—	0.99	0.25
5178A	0.21	1.11	0.39	1.47	—	0.25
5179A	0.22	0.82	0.34	1.49	—	—
5179B	0.22	0.82	0.34	1.49	0.51	—
5179C	0.22	0.82	0.34	1.49	1.03	—
5180B	0.20	1.09	0.39	—	0.51	0.25
5181A	0.21	0.83	0.35	—	—	0.24
5181B	0.21	0.83	0.35	—	0.49	0.24
5181C	0.21	0.83	0.35	—	0.97	0.24
5182A	0.22	0.55	0.37	1.48	—	0.25
5183A	0.22	0.81	0.39	1.47	—	0.54
5186A	0.22	1.10	0.40	0.72	—	0.50
5187A	0.23	0.85	0.40	—	—	0.49
5187C	0.23	0.85	0.40	—	0.97	0.49
5188A	0.23	1.13	0.39	—	—	0.50
5189B	0.20	1.10	0.39	—	0.56	—
5190B	0.21	0.85	0.38	0.73	0.51	—
5191C	0.20	0.54	0.33	—	1.02	—
5194A	0.20	1.06	0.34	1.44	—	0.51
5269B	0.21	0.47	0.29	—	0.51	0.49
5269C	0.21	0.45	0.29	—	0.96	0.49
5462	0.22	0.55	0.35	1.40	0.53	—
5463A	0.22	0.80	0.35	1.40	—	0.25
5465A	0.21	1.12	0.36	—	—	—
5465C	0.22	1.11	0.36	—	1.01	—

[a]Nominal contents of 0.015 pct P, 0.02 pct S, and 0.02 to 0.08 pct Al were also present.
[b]A dash denotes <0.05 pct of the element present.

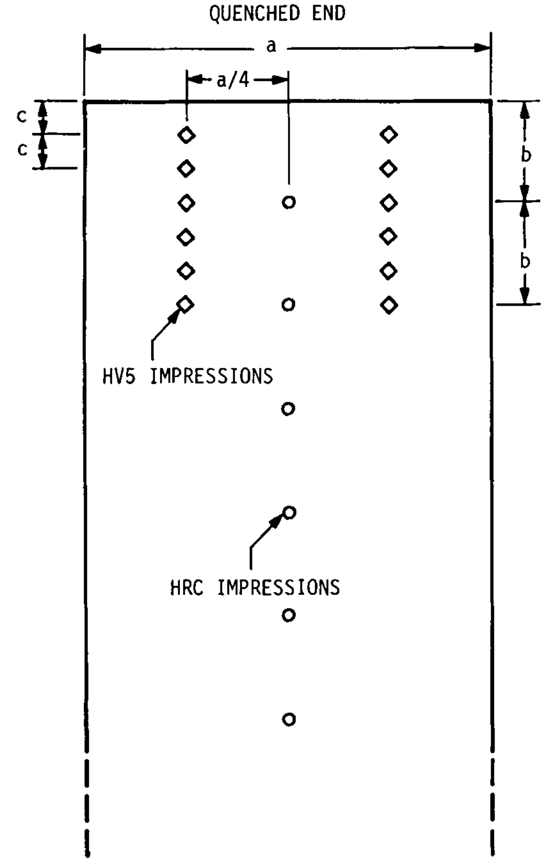
to positions of equivalent cooling rate on the Jominy bar. With these data, one need only determine experimentally at what point on the Jominy bar the steel has transformed to 50 pct martensite, and convert this J_D to D_I *via* Figure 6.

There are several methods for determining the Jominy bar location which exhibits 50 pct martensite. One is by metallographic examination. However, in the low carbon materials studied here, the difference in appearance between the martensite and bainite, the predominant non-martensitic transformation product when the material is "50 pct hardened," is not pronounced. This makes detection of the 50 pct martensite location by optical metallography difficult, time consuming, and of questionable accuracy. It was therefore decided to use another method of detection, one relying on hardness.

Hodge and Orehoski[13] have correlated hardness as a function of carbon content for steels quenched to various amounts of martensite. Figure 7 presents their data for fully martensitic and 50 pct martensitic materials. The fully martensitic curve shown in Figure 7 represents the hardness that Hodge and Orehoski obtained[13] at the J_1 position. (It is important to note that this hardness was 0 to 2 HRC higher than the 99.9 pct martensite hardness determined by them at $J_D > J_1$.) In principle, one could determine the position at which the hardness corresponds to 50 pct martensite for the carbon content in question from Figure 7, and determine D_I from these values by taking the Jominy position exhibiting this hardness and the data of Figure 6.

In this study, however, the hardness method just described was modified before being used to determine D_I. The curves of Figure 7 were used to relate the 50 pct martensite hardness to the quenched-end hardness* as measured on the Jominy bar, rather than to the carbon content determined by chemical analysis. Thus, if the quenched-end hardness was found to be 45 HRC (Point "A" on Figure 7), the 50 pct martensite hardness was taken as 32.5 HRC (Point "B" on Figure 7). The reason for using a drop in hardness rather than an absolute hardness value as the measure of 50 pct martensite is discussed below.

*The present authors have assumed that the fully martensitic hardness values shown in Figure 7, obtained by Hodge and Orehoski at the J_1 position,[13] are equivalent to those that would have been obtained right at the quenched end (IH). Since 10 out of the 35 steels that they examined possessed an identical distance hardness at the J_1 and J_2 positions and only three steels had a D_I value less than 38.1 mm (1.5 in.) where a maximum difference of about 0.8 HRC would be expected between IH and DH at J_1, this assumption is considered valid. Vickers hardness measurements between IH and J_1 were used for determining the fully martensitic hardness of the steels in this study because, unlike Hodge and Orehoski's steels, a number of the steels shown in Table I had insufficient hardenability to assume that $IH = DH$ at J_1.



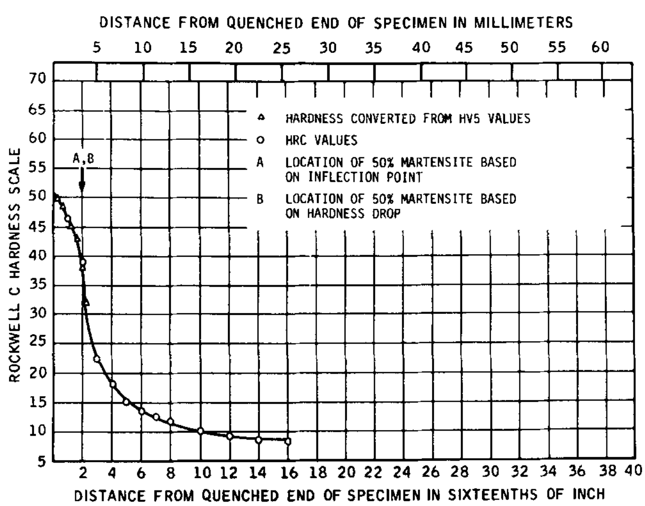
DIMENSIONS:

a = Flat Width, 8.5 mm (0.33 in.)

b = Rockwell Impression Spacing, 1.59 mm (0.063 in.)

c = Vickers Impression Spacing, 0.53 mm (0.021 in.)

Fig. 3—Schematic illustration of hardness impressions taken on the Jominy bar flats.

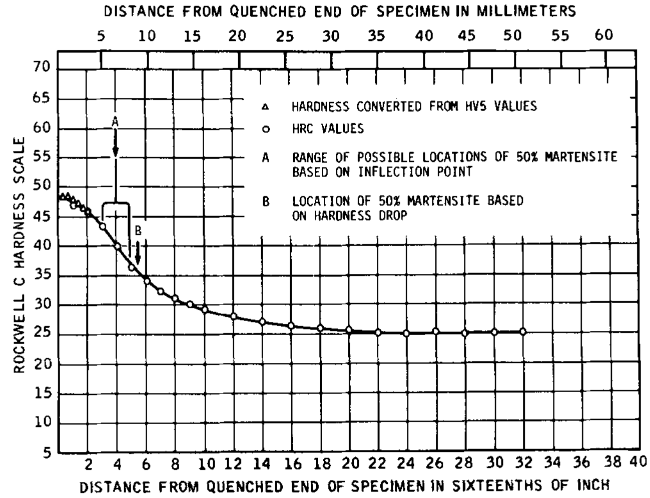


HEAT 5164A: 0.22%C, 0.56%Mn, 1.45%Ni

ASTM GRAIN SIZE No. 9

Fig. 4—Jominy curve for a steel of low hardenability.

From the Jominy curves, IH/DH ratios were determined at the Jominy distances J_1, J_2, J_3, J_4, J_6, J_8, J_{10}, J_{12}, and J_{16}. The quenched-end hardness (IH) was determined by extrapolation of the hardness data to J_0. For each J_D, a



HEAT 5181C: 0.21%C, 0.83%Mn, 0.97%Cr, 0.24%Mo

ASTM GRAIN SIZE No. 9

Fig. 5—Jominy curve for a steel of medium hardenability.

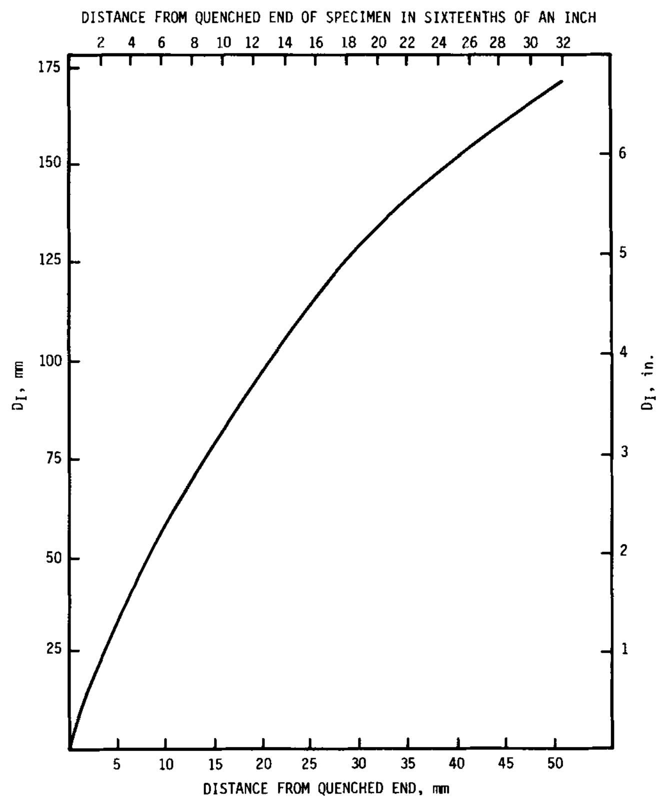


Fig. 6—Positions along the Jominy bar having the same cooling rate as the centers of cylinders of various diameters subjected to an ideal quench (Ref. 12).

tabulation of IH/DH vs the experimentally determined D_I values was then constructed. A computer was used to fit a polynomial expression of the form

$$IH/DH\Big|_{J_D=\text{const.}} = \sum_{l=0}^{n} \alpha_l D_I^l \qquad [1A]$$

to these data, with the value of n (order of the polynomial expression) being chosen in accordance with the criteria discussed below. The analytical expressions thereby obtained for each J_D were then used to calculate and tabulate IH/DH values at regular intervals of D_I.

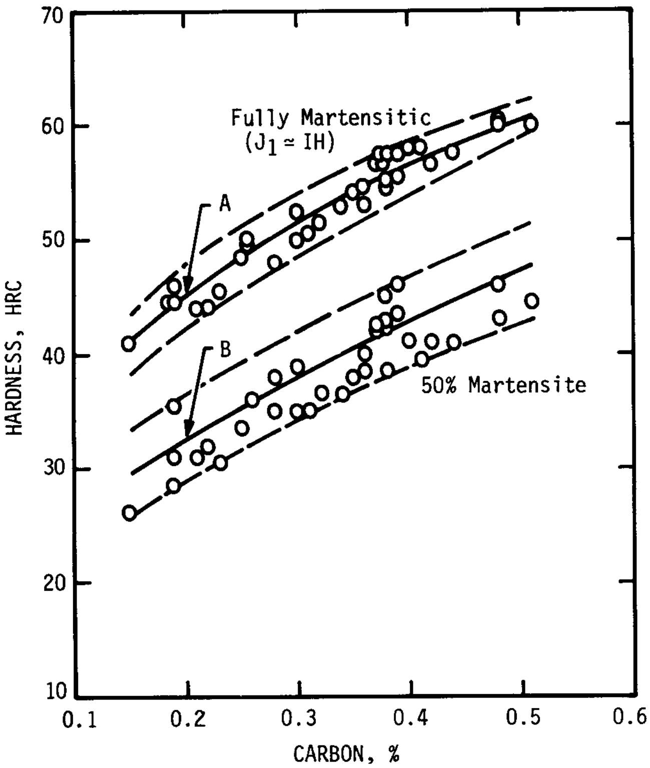


Fig. 7 — Hardness as a function of carbon content (Ref. 13).

Most of the existing literature on hardenability and the ideal critical diameter concept reports D_I in inches rather than SI units. For this reason, and because the aim of the entire sequence of steps described herein is to calculate an end-quench hardenability curve from composition rather than to obtain the intermediate D_I values, the analytical expressions which involve D_I developed by the present authors (Eqs. [1] and [3]; see also Tables II and IV) are based on D_I values in inches rather than SI units.

IV. RESULTS AND DISCUSSION

A. *Hardness Drop as a Criterion for Dertermining D_I Experimentally*

The above described choice of a hardness drop rather than an absolute hardness value for determining the location on the Jominy bar corresponding to 50 pct martensite stems from doubt in the authors' minds regarding the accuracy of the data in Figure 7. The solid curves of Figure 7 represent the midpoints of the hardness *vs* carbon content scatter bands determined from the data of Hodge and Orehoski.[13] It is clear that the fully martensite hardness for a given carbon content is not well defined, and the 50 pct martensite hardness even less well defined. Thus, it was decided here not to use an absolute hardness value to determine the 50 pct martensite position on the Jominy bar.

It was thought, however, that a given hardness value relative to the quenched-end hardness could be used to locate the 50 pct martensite position, as described above under experimental procedure. In the Hodge and Orehoski work, the fully martensitic hardness and the 50 pct martensitic hardness for a given carbon content were determined on the same piece of steel, at different positions along a Jominy bar. Thus, regardless of any possible inaccuracies in chemical analysis, the Hodge and Orehoski data do indicate a relationship between the fully martensitic hardness and the 50 pct martensitic hardness, which can be expressed as the vertical distance between the two curves of Figure 7. If the fully martensitic hardness is A (shown in Figure 7), at the corresponding carbon content the 50 pct martensitic hardness is B, *i.e.*, a drop of 12.5 HRC at a nominal carbon content of 0.2 pct. It is because of this relationship, and the aforementioned uncertainty in the absolute hardness values, that relative rather than absolute hardness values were used here to determine the 50 pct martensite location on the Jominy bar.

It should be mentioned that another method exists for determining the 50 pct martensite position from hardness. This commonly used method, often called the inflection point method, is based on the work of Grossmann[2,3] who found that for his medium carbon steels there was an excellent correlation between the 50 pct martensite position, as measured metallographically, and the position of most rapid hardness change (inflection point) on the Jominy bar. For steels with low hardenability, the inflection point method works quite well and will yield consistent results. For the steep hardness profile shown in Figure 4, obtained from a steel with low hardenability, the inflection point is clearly very close to J_2. However, for steels with higher hardenability which exhibit more gently sloping Jominy curves, it is more difficult to determine the exact location of the inflection point. For example, an inflection point can be chosen between J_3 and J_5 in Figure 5, depending on how the curve is drawn for this medium hardenability steel. From Figure 6, these 50 pct martensite Jominy position extremes will result in a D_I value between 33 and 50 mm (1.30 and 1.95 inches), a range that is far too wide to yield consistent results. For flat Jominy curves obtained from steels with high hardenability, such as the one shown in Figure 8 for a commercial heat of SAE EX55 steel, it is virtually impossible to determine the 50 pct martensite position using the inflection point method.

Locating the distance to a given drop in hardness has a much lower uncertainty; this uncertainty is usually a maximum of one J_D unit (the distance between two successive

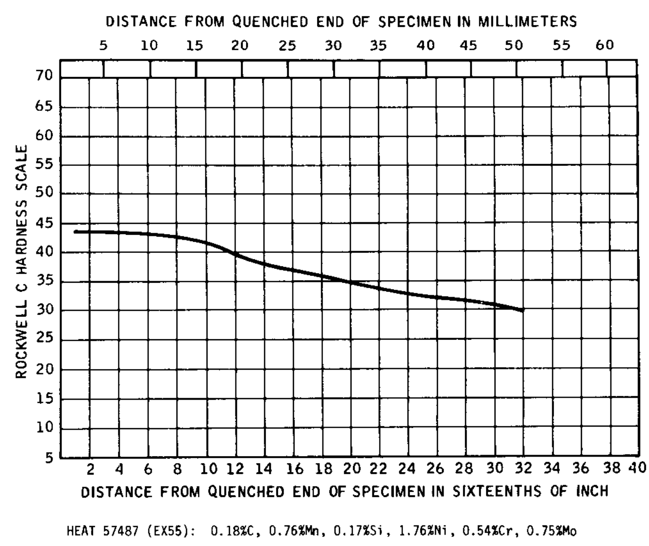


Fig. 8 — Jominy curve for a steel of high hardenability.

hardness measurements). Because of this, it was decided to do all analytical work on the IH/DH ratios using the D_I values determined by the hardness drop method rather than the inflection point method.

B. *Form of the IH/DH vs D_I Curves and Comparison with Other Work*

Figure 9(a) presents a family of IH/DH *vs* D_I curves. There are nine plots of the analytical expressions obtained by fitting a polynomial (Eq. [1A]) to the measurements of IH/DH *vs* D_I. The functions plotted are those polynomials which gave the best fit to the data based on the following criteria: (1) high multiple correlation coefficient, (2) low standard error of estimate, (3) preservation of the expected hyperbolic shape to the curves with a minimum of oscillations.

As shown in Figure 9(a), especially at the higher J_D, even the selected best fitting analytical expressions still display a small amount of oscillation for $D_I > 61$ mm (2.4 inches), that is, the curves do not smoothly approach a value of 1.0 at a continuously decreasing rate. These oscillations could not be completely avoided because of the nature of the experimental data which, although fairly uniformly distributed over the range $22 < D_I < 61$ mm ($0.85 < D_I < 2.4$ inches), are nonuniformly distributed and relatively sparse at higher D_I values (see Figures 10 to 12).

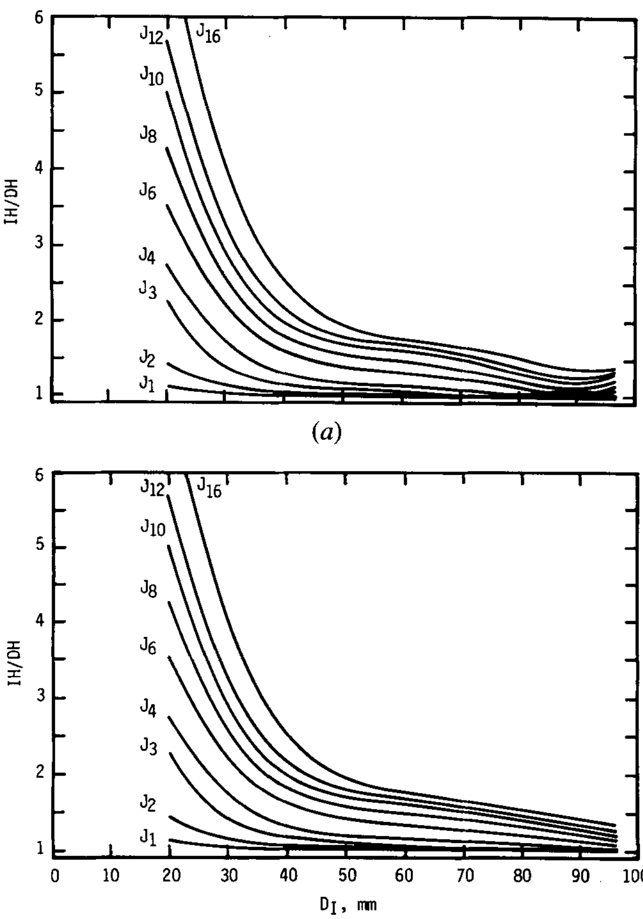


Fig. 9—Plots of analytical expressions for IH/DH *vs* D_I. (*a*) Polynomial regression curves for all data, (*b*) polynomial regression curves for $D_I < 61$ mm (2.4 in.) with linear regression for $D_I \geq 61$ mm (2.4 in.).

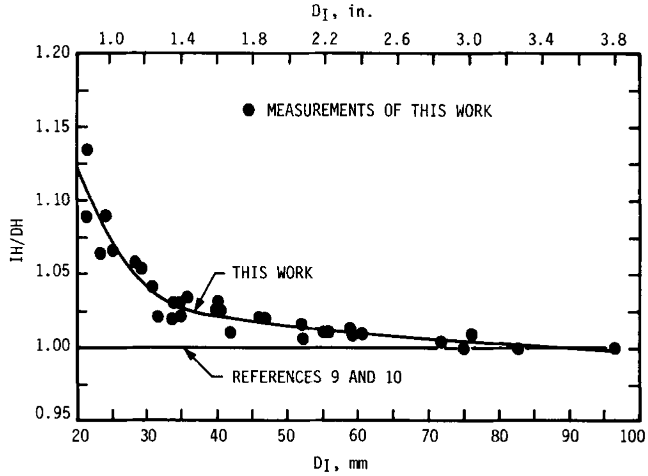


Fig. 10—Comparison of IH/DH ratios determined by various authors for J_1.

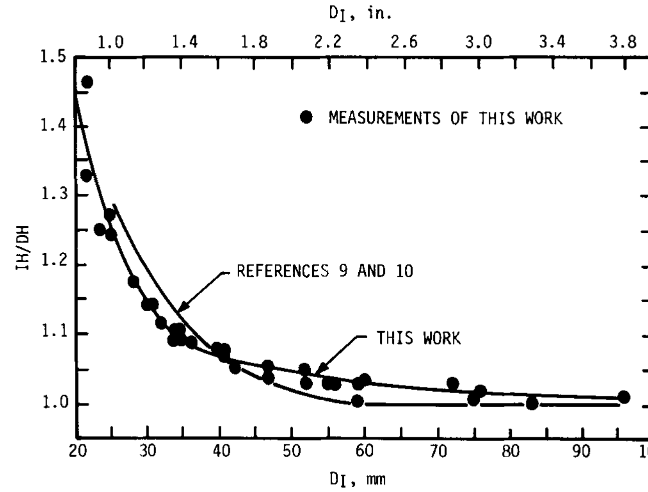


Fig. 11—Comparison of IH/DH ratios determined by various authors for J_2.

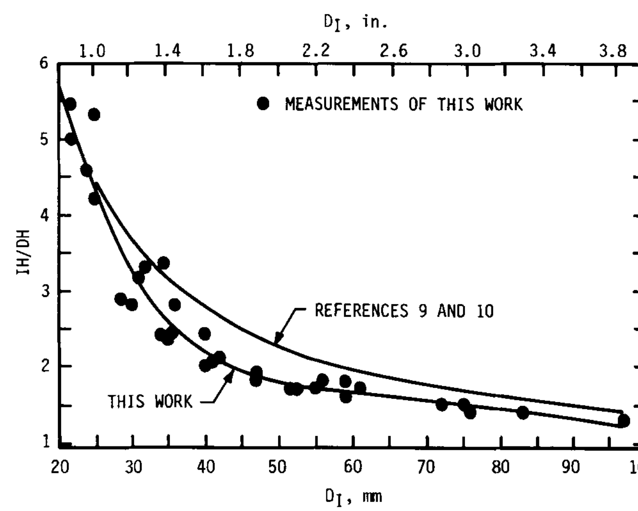


Fig. 12—Comparison of IH/DH ratios determined by various authors for J_{12}.

To eliminate these oscillations for all J_D, the following procedure was used: for $D_I < 61$ mm (2.4 inches), the polynomial expression (α_i coefficients) was used to describe the curve, and for $D_I \geq 61$ mm (2.4 inches), a straight line

obtained by linear regression (α_l' coefficients) and forced to continuity with the polynomial at $D_I = 61$ mm (2.4 inches) was used to describe the curve, with the expression

$$IH/DH|_{J_D=\text{const.}} = \begin{cases} \sum_{l=0}^{n} \alpha_l D_I^l & \text{when} \quad D_I < 61 \text{ mm (2.4 inches)} \\ \sum_{l=0}^{1} \alpha_l' D_I^l & \text{when} \quad D_I \geq 61 \text{ mm (2.4 inches)} \end{cases} \quad [1B]$$

(Examples of the resulting curves thus "smoothed out" are shown in Figures 9(b) through 12.)

The regression coefficients for the equations relating IH/DH and D_I are shown in Table II. For added convenience, the IH/DH ratios calculated from Eq. [1B] and Table II are shown in Table III for the incrementally increasing D_I.

Figures 10 through 12 are additional plots of the analytical expressions for IH/DH as determined in this study. Also shown are the actual data points from this work and the IH/DH curves of other authors.[9,10] The analytical expressions of this work are clearly a better fit to the carburizing steel data than those previously published. This is especially true at J_1, where other authors have assumed that IH/DH is equal to unity regardless of the magnitude of D_I (see Figure 10). As shown in Table III, for 0.2 pct C steels with D_I less than 76 mm (3.0 inches), IH/DH is in fact greater than unity at J_1.

V. JOMINY HARDENABILITY CURVE CALCULATION

The following sections discuss the procedures for calculating Jominy hardenability curves as derived from the results of this and other investigations. A sample calculation is described in the appendix section of this paper.

A. *Calculation of D_I Values from Composition*

A D_I value can be calculated using the equation

$$D_I = (D_I^\circ)(MF_{\text{Si}})(MF_{\text{Mn}})(MF_{\text{Ni}})(MF_{\text{Cr}})(MF_{\text{Mo}}) \quad [2]$$

where D_I°, the base diameter, is dependent on carbon content and grain size as shown in Figure 2 and MF_x is the multiplying factor for an alloying element x, as shown in Figure 1. To facilitate computerization of D_I calculations, polynomial regression equations were fit to the curves[4,5] shown in these two figures. The regression coefficients β_l and γ_l in the model equations

$$D_I^\circ|_{GS=\text{const.}} = \sum_{l=0}^{3} \beta_l (\text{pct C})^l \quad [3]$$

and

$$MF_x = \sum_{l=0}^{n} \gamma_l (\text{pct x})^l \quad [4]$$

where pct C is carbon content and pct x is another alloying element (Si, Mn, Cr, Ni, Mo), and are shown in Tables IV and V, respectively. (It should be mentioned that other β_l and γ_l coefficients have been published recently for carbon contents up to 1.25 pct.[14] Since these authors[14] used the unrevised DeRetana and Doane alloy factors[4] and only integer grain sizes between 6 and 9 for carburizing steels with 0.15 to 0.25 pct C, the coefficients shown in Tables IV and V should result in more accurate D_I calculations.)

B. *Calculation of IH/DH Ratios*

Once the D_I value has been calculated as just described, the IH/DH ratios can be determined. For D_I values up to 97 mm (3.8 inches), the IH/DH ratios at selected Jominy positions can be calculated using the family of IH/DH equations, Eq. [1B], and the regression coefficients (α_l and α_l') shown in Table II; the IH/DH ratios shown in Table III can also be used. For D_I values greater than 97 mm (3.8 inches), which are outside the validity range of the α_l and α_l' coefficients, it is suggested that the IH/DH ratios be obtained

Table II. Regression Coefficients for *IH/DH* Ratios *vs* Ideal Critical Diameter for Selected Jominy Positions [Valid for $D_I \leq 97$ mm (3.8 Inches)]

Order (l) of D_I in Inches	Polynomial Regression Coefficients (α_l) for 20 mm (0.8 Inch) $\leq D_I <$ 61 mm (2.4 Inches) *IH/DH* Ratio at Jominy Positions ("D" in J_D):[a]								
	1	2	3	4	6	8	10	12	16
0	1.902894	4.179541	10.56074	8.252814	10.73275	13.72446	16.79252	18.48272	22.73197
1	−1.924937	−6.698973	−20.28613	−11.15557	−14.41716	−19.05215	−23.71182	−25.45963	−30.21293
2	1.66193	5.703272	17.33873	6.558123	8.347587	11.15866	13.88207	14.49685	16.34534
3	−0.7052759	−2.391192	− 7.304336	− 1.696229	− 2.140781	− 2.883596	− 3.576242	− 3.3645426	− 3.914098
4	0.1457077	0.4893308	1.503764	0.1610797	0.2020699	0.2733759	0.3375566	0.3371358	0.3463339
5	−0.01169816	−0.03898436	− 0.1206152	0.0	0.0	0.0	0.0	0.0	0.0
Order (l) of D_I in Inches	Linear Regression Coefficients (α'_l) for 61 mm (2.4 Inches) $\leq D_I \leq$ 97 mm (3.8 Inches) *IH/DH* Ratio at Jominy Positions ("D" in J_D):								
	1	2	3	4	6	8	10	12	16
0	1.027047	1.063242	1.113486	1.368053	1.731265	2.050338	2.296882	2.379248	2.449352
1	−0.007535958	−0.01471938	−0.02356409	−0.09123124	−0.1703201	−0.2384788	−0.2888208	−0.29558	−0.2912418

[a]Distance in sixteenths of an inch from the quenched end. Since the polynomials are hyperbolic in form, as shown in Figure 9, roundoff of the coefficients could result in large inaccuracies. (The coefficients were calculated to eight significant figures in computer exponential notation; trailing zeros are not included in this table.)

Table III. IH/DH Ratios Calculated for Various Jominy Distances (J_D) in Carburizing Steels of Different D_I

D_I, in. (mm)	IH/DH J_1	J_2	J_3	J_4	J_6	J_8	J_{10}	J_{12}	J_{16}
0.8 (20.3)	1.12	1.43	2.27	2.72	3.53	4.26	5.01	5.66	7.16
0.9 (22.9)	1.09	1.32	1.94	2.39	3.09	3.69	4.31	4.88	6.15
1.0 (25.4)	1.07	1.24	1.69	2.12	2.72	3.22	3.72	4.21	5.30
1.1 (27.9)	1.05	1.18	1.51	1.90	2.42	2.83	3.24	3.66	4.57
1.2 (30.5)	1.04	1.14	1.38	1.71	2.17	2.51	2.85	3.21	3.97
1.3 (33.0)	1.03	1.11	1.29	1.57	1.97	2.26	2.53	2.84	3.47
1.4 (35.6)	1.03	1.09	1.23	1.45	1.81	2.06	2.29	2.55	3.06
1.5 (38.1)	1.02	1.07	1.19	1.37	1.69	1.91	2.10	2.31	2.73
1.6 (40.6)	1.02	1.07	1.16	1.30	1.59	1.79	1.96	2.14	2.47
1.7 (43.2)	1.02	1.06	1.14	1.25	1.52	1.70	1.85	2.00	2.27
1.8 (45.7)	1.02	1.05	1.13	1.22	1.46	1.64	1.78	1.90	2.12
1.9 (48.3)	1.02	1.05	1.12	1.20	1.42	1.59	1.72	1.83	2.00
2.0 (50.8)	1.02	1.05	1.11	1.18	1.40	1.56	1.69	1.78	1.92
2.1 (53.3)	1.01	1.04	1.10	1.17	1.37	1.54	1.66	1.74	1.85
2.2 (55.9)	1.01	1.04	1.08	1.16	1.36	1.52	1.64	1.72	1.81
2.3 (58.4)	1.01	1.03	1.07	1.16	1.34	1.50	1.63	1.69	1.78
2.4 (61.0)	1.01	1.03	1.06	1.15	1.32	1.48	1.61	1.67	1.75
2.5 (63.5)	1.01	1.03	1.05	1.14	1.31	1.45	1.57	1.64	1.72
2.6 (66.0)	1.01	1.02	1.05	1.13	1.29	1.43	1.55	1.61	1.69
2.7 (68.6)	1.01	1.02	1.05	1.12	1.27	1.41	1.52	1.58	1.66
2.8 (71.1)	1.01	1.02	1.05	1.11	1.25	1.38	1.49	1.55	1.63
2.9 (73.7)	1.01	1.02	1.05	1.10	1.24	1.36	1.46	1.52	1.60
3.0 (76.2)	1.00	1.02	1.04	1.09	1.22	1.33	1.43	1.49	1.58
3.1 (78.7)	1.00	1.02	1.04	1.09	1.20	1.31	1.40	1.46	1.55
3.2 (81.3)	1.00	1.02	1.04	1.08	1.19	1.29	1.37	1.43	1.52
3.3 (83.8)	1.00	1.01	1.04	1.07	1.17	1.26	1.34	1.40	1.49
3.4 (86.4)	1.00	1.01	1.03	1.06	1.15	1.24	1.31	1.37	1.46
3.5 (88.9)	1.00	1.01	1.03	1.05	1.14	1.22	1.29	1.34	1.43
3.6 (91.4)	1.00	1.01	1.03	1.04	1.12	1.19	1.26	1.32	1.40
3.7 (94.0)	1.00	1.01	1.03	1.03	1.10	1.17	1.23	1.29	1.37
3.8 (96.5)	1.00	1.01	1.02	1.02	1.08	1.14	1.20	1.26	1.34

Table IV. Polynomial Regression Coefficients for Base Ideal Critical Diameter and Initial Hardness *vs* Carbon Content and Grain Size (Valid for 0.15 to 0.5 Pct C)

Order (l) of Pct C	Regression Coefficients (β_l) for Base Ideal Critical Diameter (D_I in inches) Corresponding to ASTM Grain Size Numbers of:[a] 4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0	Regression Coefficients[b] (δ_l) for Initial Hardness (IH), HRC
0	0.14781	0.13619	0.12622	0.09662	0.08942	0.08718	0.08647	0.10764	0.10075	0.08572	0.07217	0.07577	0.07796	27.68534
1	1.4980	1.4847	1.4575	1.6371	1.5654	1.5073	1.4386	1.1788	1.2181	1.2735	1.2937	1.1896	1.0891	96.30058
2	1.2265	1.0976	1.0013	0.40652	0.62486	0.67069	0.7171	1.2501	0.81968	0.60656	0.54017	0.73246	0.92708	−60.07454
3	−2.8436	−2.6331	−2.4508	−1.9360	−2.2363	−2.2278	−2.2290	−2.6466	−2.0419	−1.8676	−1.8609	−2.0339	−2.2220	0.0

[a]D_I° regression coefficients were obtained from the curves[4,5] shown in Figure 2. Half grain size coefficients were interpolated.
[b]IH coefficients were obtained from polynomial fits of the actual data points shown in Figure 7.

Table V. Polynomial Regression Coefficients for Alloy Multiplying Factors *vs* Alloy Content (Valid for 0.15 to 0.25 Pct C)

Order (l) of Pct x	Regression Coefficients (γ_l) for the Multiplying Factors (MF_x) Corresponding to the Elements (x)[a] Si		Mn	Cr	Ni	Mo	
0	1.0	3.0730	1.0405	1.0095	0.98962	1.0027	1.0424
1	0.0	−6.6649	0.31563	−0.23412	0.31594	1.0805	1.1227
2	0.0	7.6391	2.8571	3.6325	0.03394	1.3081	4.6832
3	0.0	−3.5786	−2.0288	−3.7847	−0.05030	−0.35477	−4.8030
4	0.0	0.61992	0.54441	1.4935	−0.01746	0.0	1.7706
Range(s) of Validity	0.6 pct Si max	0.6 < pct Si ≤ 2.0	1.4 pct Mn max	1.5 pct Cr max	3.75 pct Ni max	0.75 pct Ni max; 1.0 pct Mo max	0.75 < pct Ni ≤ 3.75; 1.0 pct Mo max

[a]Multiplying factor regression coefficients were obtained from the curves[5] shown in Figure 1.

from the work of Sponzilli *et al.*,[10] reproduced in Table VI for D_I values between 97 and 152 mm (3.8 and 6.0 inches).

C. *Calculation of a Jominy Hardenability Curve*

A polynomial regression equation relating initial hardness (*IH*) and carbon content, *i.e.*,

$$IH = \sum_{l=0}^{2} \delta_l (\text{pct C})^l \qquad [5]$$

was fit to the actual data shown in the uppermost band of Figure 7. The coefficients (δ_l) for Eq. [5] are shown in Table IV. The distance hardness (*DH*) is obtained at each Jominy position by dividing the *IH* value calculated from Eq. [5] by the appropriate *IH*/*DH* ratio. It should be mentioned that if the user has a measured value of *IH*, *i.e.*, the *fully martensitic* hardness of the steel in question, he can use that value rather than calculating *IH* using the Hodge and Orehoski J_1 data (Eq. [5]) as described above.

D. *Residual Analysis*

Since one of the purposes of this work was to provide an improved method for calculating the Jominy curves of carburizing steels (0.15 to 0.25 pct C), some comparison of the method developed here with another method is appropriate. The alternate method selected for comparison is that developed by Just.[15] The equation chosen from Just's work[15] is the one explicitly developed for carburizing steels (C < 0.28 pct), namely,

$$DH \Big|_{J_4 \text{ to } J_{25}} = 87\text{C} + 14\text{Cr} + 5.3\text{Ni} + 29\text{Mo} + 16\text{Mn} - 21.2\sqrt{E} + 2.21E + 22 \qquad [6]$$

where E is the distance (in 1/16 inch) from the quenched end. Selection of the Just method was based on the complimentary comments of its users in the discussion section of the review by Doane,[7] its easy availability in the literature,[15] and its specific use of carburizing steels during the development of Eq. [6].

The data set used for comparing the two methods was composed of 24 Climax laboratory heats as briefly described in Table VII. These heats are independent of those shown in Table I, *i.e.*, the data set described in Table VII was not used to develop any of the equations contained in this work. The independent data set was chosen so that the *IH*/*DH* coefficients shown in Table II could be used, that is, the maximum calculated D_I value was less than 97 mm (3.8 inches).

Table VII. Summary Statistics Describing the 24 Climax Compositions Used in the Residual Analysis

Quantity	Minimum Value	Maximum Value	Mean[a]	Median[a]
C[b]	0.12	0.30	0.21	0.20
Mn	0.58	1.29	0.94	0.87
Si	0.23	0.43	0.28	0.28
Ni[c]	0	1.14	0.12	0
Cr	0	1.23	0.79	0.89
Mo	0	0.49	0.17	0.2
Grain Size No.[d]	7.0	10.0	8.9	9.3
D_I Calc.[e] mm (in.)	33 (1.3)	86 (3.4)	53 (2.1)	48 (1.9)

[a]Mean = sum of the measurements divided by the number of measurements.
Median = value of the measurements that is in the middle when the measurements are arranged in order of magnitude.
[b]Only one heat with C < 0.17 and one heat with C > 0.28.
[c]Only four heats with Ni > 0 and one heat with Ni > 0.55.
[d]Only four heats with grain sizes less than 8.5.
[e]Calculated values using the Climax multiplying factor method discussed in this paper.

Table VI. Initial Hardness/Distance Hardness (*IH*/*DH*) Ratios[a]

Ideal Critical Diameter (D_I), In.	Jominy Distance (J_D)																			
	J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8	J_9	J_{10}	J_{11}	J_{12}	J_{13}	J_{14}	J_{15}	J_{16}	J_{20}	J_{24}	J_{28}	J_{32}
3.8	1.0	1.0	1.01	1.05	1.10	1.14	1.19	1.23	1.28	1.33	1.38	1.43	1.48	1.53	1.59	1.65	1.80	1.88	1.94	2.00
3.9	1.0	1.0	1.01	1.05	1.09	1.14	1.18	1.22	1.27	1.31	1.36	1.41	1.46	1.51	1.56	1.62	1.76	1.84	1.90	1.96
4.0	1.0	1.0	1.0	1.04	1.08	1.12	1.16	1.20	1.25	1.29	1.33	1.38	1.43	1.48	1.53	1.59	1.72	1.80	1.86	1.92
4.1	1.0	1.0	1.0	1.04	1.08	1.11	1.15	1.18	1.23	1.28	1.32	1.36	1.41	1.46	1.51	1.56	1.68	1.77	1.82	1.88
4.2	1.0	1.0	1.0	1.03	1.06	1.10	1.14	1.17	1.22	1.25	1.29	1.34	1.38	1.42	1.48	1.53	1.65	1.73	1.78	1.84
4.3	1.0	1.0	1.0	1.03	1.06	1.10	1.13	1.16	1.20	1.24	1.28	1.32	1.37	1.41	1.46	1.50	1.62	1.70	1.75	1.80
4.4	1.0	1.0	1.0	1.02	1.05	1.09	1.12	1.15	1.19	1.23	1.27	1.30	1.35	1.39	1.43	1.47	1.58	1.66	1.72	1.76
4.5	1.0	1.0	1.0	1.02	1.05	1.08	1.11	1.14	1.18	1.22	1.25	1.28	1.32	1.36	1.40	1.44	1.55	1.63	1.68	1.73
4.6	1.0	1.0	1.0	1.02	1.05	1.07	1.10	1.12	1.15	1.19	1.22	1.26	1.29	1.33	1.37	1.41	1.52	1.59	1.64	1.69
4.7	1.0	1.0	1.0	1.01	1.04	1.06	1.09	1.11	1.14	1.17	1.20	1.24	1.28	1.31	1.35	1.38	1.49	1.56	1.61	1.65
4.8	1.0	1.0	1.0	1.01	1.02	1.05	1.07	1.10	1.13	1.16	1.19	1.22	1.25	1.28	1.32	1.36	1.46	1.53	1.57	1.62
4.9	1.0	1.0	1.0	1.0	1.0	1.03	1.06	1.08	1.11	1.14	1.17	1.20	1.23	1.26	1.29	1.33	1.43	1.49	1.53	1.58
5.0	1.0	1.0	1.0	1.0	1.0	1.02	1.05	1.07	1.10	1.13	1.15	1.18	1.20	1.25	1.28	1.31	1.40	1.46	1.50	1.54
5.1	1.0	1.0	1.0	1.0	1.0	1.01	1.04	1.06	1.09	1.11	1.14	1.17	1.19	1.22	1.25	1.28	1.37	1.43	1.47	1.51
5.2	1.0	1.0	1.0	1.0	1.0	1.0	1.03	1.05	1.08	1.10	1.13	1.15	1.18	1.20	1.23	1.25	1.34	1.39	1.43	1.47
5.3	1.0	1.0	1.0	1.0	1.0	1.0	1.02	1.04	1.06	1.09	1.11	1.13	1.16	1.18	1.21	1.23	1.31	1.36	1.39	1.43
5.4	1.0	1.0	1.0	1.0	1.0	1.0	1.02	1.03	1.05	1.08	1.11	1.12	1.15	1.18	1.20	1.21	1.28	1.33	1.36	1.40
5.5	1.0	1.0	1.0	1.0	1.0	1.0	1.01	1.03	1.05	1.07	1.08	1.10	1.12	1.13	1.15	1.18	1.25	1.29	1.33	1.37
5.6	1.0	1.0	1.0	1.0	1.0	1.0	1.01	1.02	1.04	1.05	1.07	1.09	1.10	1.12	1.14	1.16	1.22	1.26	1.28	1.33
5.7	1.0	1.0	1.0	1.0	1.0	1.0	1.01	1.02	1.03	1.04	1.06	1.07	1.09	1.10	1.11	1.13	1.19	1.23	1.25	1.29
5.8	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.01	1.02	1.04	1.05	1.06	1.07	1.09	1.10	1.11	1.17	1.19	1.22	1.25
5.9	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.01	1.01	1.02	1.03	1.04	1.05	1.06	1.08	1.09	1.13	1.16	1.18	1.21
6.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.01	1.01	1.02	1.03	1.04	1.05	1.06	1.07	1.10	1.13	1.15	1.18

[a]From Reference 10

Residuals were calculated for the two prediction methods by subtracting the predicted distance hardness from the distance hardness measured experimentally for the 24 independent compositions. The absolute values of these residuals were also considered; this latter calculation is a more severe test than simply calculating a residual since overestimates of distance hardness will not cancel the effects of underestimates of distance hardness in determining measures of central tendency (mean and median).

The results of the residual analysis are shown in Table VIII for selected Jominy positions. Both methods resulted in good agreement between experimental and predicted distance hardness. The present work showed slightly better agreement with experiment than Just's method; in general, the minimum, maximum, mean, and median residuals were closer to zero for the method developed in this work. Both methods tended to overestimate the Jominy distance hardness, *i.e.*, exhibit negative mean and median residuals.

The simplicity of Eq. [6] and the comparable accuracy of its predictions would make Just's method of calculating the end-quench hardenability curve preferable to that developed in this work, were it not for one shortcoming. Just's method for carburizing steels does not permit the calculation of hardness at $J_D < J_4$. As illustrated in Figure 4, substantial changes in hardness occur in this region of the hardenability bar for low to medium hardenability carburizing steels, which comprise the bulk of carburizing steels used commercially. Even for a high hardenability carburizing steel such as SAE 4817, the hardness at J_4 will be about 10 pct below the quenched-end hardness.

In addition, it should be noted that the data set used here for comparing the two methods does not provide a very severe test of Eq. [6]. As shown in Table VII, 20 of the 24 steels examined had grain sizes that were within a range of 1.5 ASTM Grain Size Numbers, and only one of the steels had a nickel content in excess of 0.55 pct. The influence of grain size on hardenability is well documented, as is a synergism between nickel and molybdenum at nickel contents in excess of 0.75 pct.[5] Equation [6] does not account for either of these effects. Had a different data set been used for comparing the two prediction methods, Eq. [6] probably would have yielded substantially higher residuals than the prediction method developed in the present work. The authors believe, therefore, that the method developed here will generally provide more useful information for carburizing steels than Eq. [6].

Jominy curves determined experimentally, and those predicted using the method developed in this paper, are shown for three steels in Figure 13; experimental values for D_I and grain size were determined by the hardness drop and linear intercept methods, respectively. These steels were not used to develop the regression coefficients. Predicted Jominy curves are shown for two grain size values for each steel; a difference of half an ASTM grain size number results in a *DH* difference of 1 HRC at most. The agreement shown in Figure 13 is fairly representative of what can be expected for steels with 0.15 to 0.25 pct C when D_I is less than 97 mm (3.8 inches).

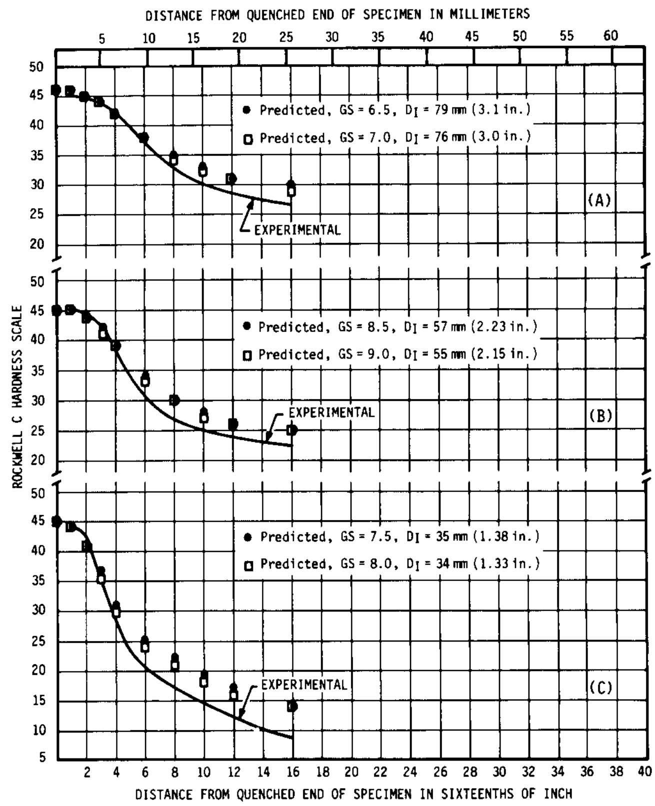


Fig. 13—Experimental and predicted Jominy curves for three steels. (*a*) Low Si-18CrMo4 steel (0.22C-0.13Si-0.9Mn-1.04Cr-0.24Mo) with measured D_I = 68.6 mm (2.7 in.) and measured grain size = 6.5 to 6.9. (*b*) Mo-modified SAE 8620 steel (0.21C-0.24Si-0.82Mn-0.55Ni-0.5Cr-0.38Mo) with measured D_I = 48.8 mm (1.92 in.) and measured grain size = 8.5 to 8.9. (*c*) SAE 4118 steel (0.21C-0.21Si-0.79Mn-0.5Cr-0.09Mo) with measured D_I = 34.8 mm (1.37 in.) and measured grain size = 7.6 to 7.9.

Table VIII. Summary Statistics Describing the Residuals Calculated Using Two Prediction Methods[a]

Jominy Position	Minimum Residuals		Maximum Residuals		Mean Residuals		Median Residuals		Mean of Absolute Values of Residuals		Median of Absolute Values of Residuals	
("D" in J_D)	This Work	Just	This Work	Just	This Work	Just	This Work	Just	This Work	Just	This Work	Just
1	−1.3	—	1.7	—	0.3	—	0.3	—	0.7	—	0.6	—
2	−1.5	—	2.5	—	1.2	—	1.2	—	1.3	—	1.2	—
3	−1.8	—	3.6	—	0.8	—	0.8	—	1.3	—	1.1	—
4	−5.2	−8.5	4.6	3.7	−0.1	−1.0	0.0	1.5	1.9	2.4	1.8	2.3
6	−5.4	−7.6	3.9	4.1	−0.7	−2.1	−1.0	−2.4	2.2	3.1	2.0	3.4
8	−4.6	−6.6	3.7	4.1	−0.8	−1.8	−0.7	−1.9	2.0	2.7	2.1	2.5
10	−4.3	−5.8	3.3	4.1	−0.7	−1.2	−0.2	−0.9	1.8	2.0	1.5	1.2
12	−4.2	−5.3	2.5	3.8	−0.8	−0.9	−0.4	−0.6	1.6	1.7	0.9	0.9
16	−4.9	−4.5	2.6	4.3	−0.9	−0.6	−0.6	−0.8	2.0	1.7	2.0	1.5

[a]Mean = sum of the measurements divided by the number of measurements.
Median = value of the measurements that is in the middle when the measurements are arranged in order of magnitude.
Residual = *DH* (experimental) − *DH* (calculated).

VI. SUMMARY

The hardness drop method for determining ideal critical diameters from experimental Jominy hardenability curves is presented and discussed. Polynomial regression equations relating Jominy distance hardness to composition and ideal critical diameter, as well as regression equations relating ideal critical diameter to composition, were developed using the hardness drop method and multiplying factors from the literature. This Jominy curve prediction method is best applied for boron-free carburizing steels containing 0.15 to 0.25 pct C. The prediction method is outlined in the appendix section of this paper, and an example calculation is shown for SAE 8620H steel.

A computer program containing the AMAX hardenability prediction system discussed in this paper has been written for mainframes and selected micro-computers. The program can be obtained, free of charge, by contacting J. M. Tartaglia.

APPENDIX

Example Jominy curve calculation

1. Calculate D_I using Eq. [2]. Use the data contained in Figures 1 and 2 or the coefficients in Tables IV and V and Eqs. [3] and [4]. Example: SAE 8620H steel with ASTM Grain Size No. 7, 0.2 pct C, 0.25 pct Si, 0.8 pct Mn, 0.5 pct Cr, 0.55 pct Ni, and 0.2 pct Mo.

$D_I^\circ = 0.384$
$MF_{Si} = 1.000$
$MF_{Mn} = 2.307$
$MF_{Cr} = 1.421$
$MF_{Ni} = 1.167$
$MF_{Mo} = 1.266$

$D_I = (0.384)\ (1.000)\ (2.307)\ (1.421)\ (1.167)\ (1.266) = 1.86$ in.

2. Determine the IH/DH ratio at each Jominy position using the D_I value calculated in Step 1.

(a) For steels with D_I less than or equal to 97 mm (3.8 inches), use the data shown in Table III, or the coefficients in Table II and Eq. [1B]. For the SAE 8620H steel with D_I = 47.3 mm (1.86 inches) calculated in Step 1 above, IH/DH values of 1.017, 1.052, 1.124, 1.205, 1.439, 1.608, 1.743, 1.858, and 2.043 were calculated for Jominy positions 1, 2, 3, 4, 6, 8, 10, 12, and 16.

(b) For steels with D_I between 97 and 152 mm (3.8 and 6.0 inches), use the data shown in Table VI.

3. Determine IH from the carbon content using the data shown in Figure 7 or the regression coefficients shown in Table IV and Eq. [5]. For the SAE 8620H example, an IH value of 45 HRC was calculated (at the 0.2 pct C content).

4. Divide IH by the appropriate IH/DH ratio obtained in Step 2 to obtain DH for each Jominy position of interest. For the SAE 8620H example, DH values of 44, 42, 40, 37, 31, 28, 26, 24, and 22 HRC were calculated for Jominy positions 1, 2, 3, 4, 6, 8, 10, 12, and 16. As shown in Figure A-1, the calculated Jominy curve for the midrange composition example falls within the specification for SAE 8620H steel.

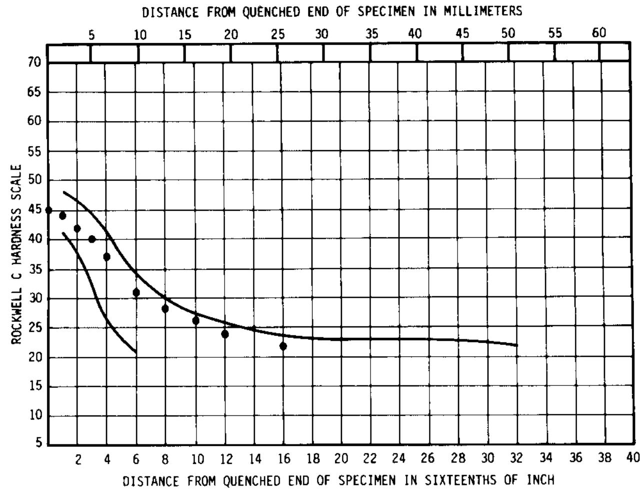


Fig. A-1 — Hardenability band specification for SAE 8620H steel. The data points were calculated for a midrange composition with 0.2 pct C, 0.25 pct Si, 0.8 pct Mn, 0.5 pct Cr, 0.55 pct Ni, and 0.2 pct Mo assuming a grain size of 7.

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